# Comparison of Voting Procedures Using Models of Electoral Competition with Endogenous Candidacy

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## 1 Introduction

This article surveys the latest advances in the literature on the comparative properties of different voting procedures in the context of political elections. In particular, we focus on the various attempts at understanding how different voting procedures affect the number of candidates and the positions they adopt. In public debate as well as academic scholarship, the comparative properties of different voting procedures has been a topic of enduring interest. This interest dates as far back as the late eighteenth century French philosopher-mathematicians Condorcet and Borda (and even earlier with the works of Lull and Cusanus), building up to the classic works of Arrow, May and Gibbard-Satthertwaite in the mid-twentieth century. The work in

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this tradition is largely confined to a rather abstract treatment of voting procedures, over an exogenously given set of alternatives.

The works of Harold Hotelling and Anthony Downs embed voting into a larger game, one of electoral competition, thereby enabling the study of how a voting procedure affects the policy positions adopted by the political parties.[1] However, the canonical Hotelling-Downs model assumes two political parties/candidates. Hence, by construction, the set of choices available to the voters is restricted to having at most two elements. It was conjectured by Maurice Duverger that the number of political parties emerging is systematically related to the voting procedure used.[2] This conjecture took the form of Duverger's law, which states an empirical regularity that the plurality rule[3] favors a two party system, and Duverger's hypothesis, which suggests that the plurality runoff rule[4] and proportional representation favor a multiparty system. The literature we review here consists of analytical approaches to studying conjectures similar to those of Duverger's by modeling political competition under alternative voting procedures.

A voting procedure is comprised of three elements: the first element, *the ballot structure*, refers to the type of ballot a voter is permitted to cast (numbers of votes each voter must and is allowed to cast, whether each voter must rank-order candidates or not, and so on); the second element, *the allocation rule*, refers to the formula by which a set of election winners is determined from the ballots cast by the voters; finally, the third element, *the district magnitude*, refers to the number of candidates to be elected in an electoral district. In this paper we do not study variations in the district magnitude; we consider elections for a single office (a single district of magnitude one) and focus on variations in the first two elements of a voting procedure.[5]

While the focus of our survey is primarily on the theoretical literature, it is instructive to look at some stylized facts about elections around the world. Consistent with the theme of our survey, we will look at the stylized facts about the number of political parties and the extent of polarization of party policies in several democracies.[6] In particular, we look at data on five democracies of which

[1] See Hotelling (1929), Downs (1957).

[2] See Duverger (1954).

[3] Under the plurality rule, each voter votes for one candidate and the candidate who gets the most votes is elected.

[4] The plurality runoff rule takes several forms. In a standard form, each voter votes for one candidate. If a candidate gets a majority of votes, he is elected. Otherwise, all but the top two vote-getters are eliminated, and voters are invited to vote for one of the two candidates still standing. The one of these two candidates who then gets a majority of votes is elected.

[5] There is a small literature on models of endogenous candidacy in multi-member district elections which we briefly refer to in the concluding section.

[6] The numbers presented below are based on our own calculations on data from the comparative manifesto dataset (Volkens et al. 2015). The dataset can be accessed at https://manifestoproject.wzb.eu/

**Table 1** Effective number of parties

| Country | # of elections | *ENP* (s.d.) |
|---|---|---|
| Canada | 21 | 3.07 (0.44) |
| UK | 17 | 2.37 (0.33) |
| US | 17 | 1.99 (0.02) |
| Australia | 27 | 2.60 (0.27) |
| France | 14 | 4.31 (0.70) |

three (Canada, UK and US) use the plurality rule for parliamentary elections, while the other two (Australia and France) use runoff rules.[7]

Lets first look at the number of political parties. A standard measure of the number of parties that is used in the political science literature is the Effective Number of Parties. Table 1 below presents the average Effective Number of Parties (*ENP*) across all national parliamentary elections held between 1946 and 2013 in five democracies. The *ENP* is an adjusted measure of the number of parties that accounts for the relative strength of each party. It is a commonly used indicator of party system fragmentation (Laakso and Taagepera 1979). The *ENP* in a given election is obtained by taking the inverse of the sum of squared vote shares of the participating parties ($v_i$s); in effect it is the inverse of the *Herfindahl Index* of concentration. The precise formula is given by:

$$ENP = \frac{1}{\Sigma_{i=1}^{M} v_i^2}$$

where $M$ is the number of parties participating in the election. Thus, the *ENP* varies from 1 (when one party obtains all the votes) to $M$ (when votes are spread equally across parties).

The numbers obtained are arguably consistent with Duverger's law (except for Canada which is, and with India, a well-known exception to Duverger's law). There also appear to be mixed evidence regarding Duverger's hypothesis in that France has a greater *ENP* than any of the plurality rule democracies while Australia has an *ENP* not too different from a plurality rule democracy such as the UK.

Turning to the issue of policy polarization, in Table 2 we present for the same set of countries and time period as above, the average degree of polarization across elections. Polarization is a measure of how different the platforms of the competing parties are, along the left-right dimension.[8] A party system is said to be heavily

[7] Australia uses the alternative vote rule, an example of instant runoff rule, in elections held to elect members of the House of Representatives; France uses the plurality runoff rule in its Presidential elections and in elections held to elect members of the Assemblée Nationale.

[8] We use manually coded data from the comparative manifesto dataset (Volkens et al. 2015). In this dataset, each sentence is coded according to its theme, in particular, a left-wing theme (e.g., a sentence in favor of multiculturalism) and a right-wing theme (e.g., a sentence in favor of maintaining law and order). The left-right position of each party is calculated by taking the

**Table 2** Polarization

| Country | # of elections | Polarization (s.d.) |
|---|---|---|
| Canada | 21 | 0.10 (0.05) |
| UK | 17 | 0.15 (0.10) |
| US | 17 | 0.08 (0.06) |
| Australia | 27 | 0.16 (0.08) |
| France | 14 | 0.21 (0.08) |

polarized if all the parties are located at the extremes, while polarization is said to be absent if all the parties are located at the center (Sartori 1976). Formally, polarization is calculated as the weighted sum of squared distances between each party's position on a left-right scale and the center of gravity of the party system, which is itself the weighted average of all the parties' positions on the left-right scale (e.g., see Dalton 2008 and Lupu 2015). Thus, we have

$$Polarization = \sum_{i=1}^{M} v_i \cdot (p_i - \bar{p})^2$$

where $v_i$ is party $i$'s vote share and $p_i$ is the policy position of party $i$ on the left-right policy spectrum with the center of gravity, denoted by $\bar{p}$, which is obtained by the formula

$$\bar{p} = \sum_{i=1}^{M} v_i \cdot p_i$$

The above table shows that, on average, it is usual to see between 10 and 20 % polarization (100 % being complete polarization). This is a rather substantial degree of polarization, even in plurality rule elections[9]; moreover, elections using runoff rules appear to lead to a greater degree of polarization.

In our survey we will be reviewing the literature on endogenous candidacy models of electoral competition keeping the following questions in mind: (1) to what extent do the models of endogenous candidacy provide predictions consistent with the stylized findings presented above; (2) if and in what sense are these models an improvement over the exogenous candidacy models in explaining the stylized facts presented above. The theoretical literature on voting procedures extends beyond the comparison of the plurality and runoff rules. More generally one would like to understand the effect of allowing voters to cast multiple votes on the

difference between the number of left and right sentences, and dividing this difference by the sum of left and right sentences altogether. This measure of the left-right position varies between −1 (extreme left) to +1 (extreme right).

[9]For instance, even in the most moderate case of the US, the polarization index of 0.08 means that party manifestos have, on average, an imbalance of about 28 % of sentences towards left or right compared to the "center of gravity" of the party system.

electoral outcomes. Hence, our survey will also try to assess (3) whether endogenous candidacy models provide in general a better paradigm for comparative analysis of voting procedures (for single office) over the exogenous candidacy models.

The remainder of the paper is organized as follows: in Sect. 2 we present a unified model of electoral competition that encompasses, as special cases, models of electoral competition with exogenous candidacy as well as various types of endogenous candidacy models reviewed in the survey. Section 3 presents a model due to Cox (see Cox 1987, 1990) which serves as our benchmark for understanding the exogenous candidacy approach to studying alternative voting procedures. Section 4 begins with a discussion of the shortcoming of Cox's approach, and provides us the rationale for endogenous candidacy models. In Sect. 5 we discuss various models of electoral competition that endogenize candidacy and the conclusions they reach regarding the properties of alternative voting procedures. Section 6 discusses the findings from Sect. 5, with particular emphasis on how the comparative properties of voting procedures are influenced by the modeling choices made under different models discussed in the previous sections. Section 7 concludes with some general remarks about the endogenous candidacy approach, its shortcomings and directions for future research.

## 2 A Unifying Framework

In this section we present a canonical spatial model of electoral competition that provides a unifying framework for the different families of models reviewed in this survey.

We consider a community that must elect a policymaker to choose a policy. The policy space $X$ is a non-empty and closed interval on the real line $\mathbb{R}$, say, $X = [0, 1]$. The community consists of a continuum of voters, $\mathcal{N}$, the size of which is normalized to unity. Each voter has single-peaked preferences over the policy space $X$. We denote the (unique) ideal policy of voter $n \in \mathcal{N}$ by $x_n \in X$. Voters' ideal policies are distributed according to a continuous and strictly increasing distribution function $F$, with corresponding density function $f$ and support $[0, 1]$. We denote by $m$ the policy at which $F(m) = 1/2$, and refer to $m$ as the median voter's ideal policy. Voter $n$'s utility from policy $x$ is given by $u^n(x) = u(|x - x_n|)$, where $u : \mathbb{R}_+ \to \mathbb{R}$ is a strictly quasi-concave function and $|x - x_n|$ is the distance between policy $x$ and voter $n$'s ideal policy, $x_n$.

There is a finite set of $M \geq 2$ potential candidates, $\mathcal{M}$, who must decide whether to contest the election and, if so, at which position.[10] We denote potential candidate $i$'s decision by $e_i \in \mathcal{E}_i \subseteq X \cup \{\emptyset\}$, where $e_i = x_i \in X$ if potential candidate $i$ contests the election at position $x_i$, and $e_i = \emptyset$ if he does not run for office. There

[10]Depending on the family of models, the set of potential candidates is disjoint from the set of voters ($\mathcal{M} \cap \mathcal{N} = \emptyset$) or is a subset of the set of voters ($\mathcal{M} \subset \mathcal{N}$).

is a cost $\delta$ that a potential candidate must bear if he chooses to contest the election. Depending on the family of models, potential candidates are either Downsians or Partisans. Downsians are purely office-motivated, i.e., are interested only in their electoral performance (e.g., vote share or winning probability) and the rent they can extract when holding office. In other words, Downsians do not care about the policy outcome; rather, they treat policymaking as an instrument towards gaining office and extracting rents. By contrast, Partisans are policy-motivated, i.e., care intrinsically about the policy outcome.[11] Partisans treat policymaking not just as an instrument towards gaining office, but also as an instrument towards choosing policy.

The policymaking process is modeled as a three stage game. At the first stage, each potential candidate makes his candidacy decision. At the second stage, voters vote simultaneously over the set of candidates. In most of this review, we assume that voters are sincere, i.e., they report directly their preferences over the set of standing candidates, without taking account of the electoral prospects of the candidates. Ties are broken equiprobably. Finally, at the third stage of the game, the elected candidate chooses and implements policy. In case no candidate is running for election, a default policy $x_0$ is implemented which, to facilitate exposition, we assume provides every voter $n$ with a utility $u^n(x_0) = -\infty$.

The solution concept is pure-strategy (subgame perfect) Nash equilibrium.

## 3 The Cox Model

Cox (1987) adapts the standard Hotelling-Downsian framework to study equilibria under alternative voting procedures.[12] In this model, the set of candidates is exogenous and given by $\mathcal{M}$. Candidates compete by simultaneously choosing their position on $X$. The set of feasible actions for each candidate $i$ is thus given by $\mathcal{E}_i = X$, where $e_i \in X$ denotes candidate $i$'s position and $e = (e_1, \ldots, e_M)$ denotes the profile of candidacy decisions.[13] Candidates are Downsians; when choosing his position, each candidate $i$ seeks to maximize his vote share $v_i(e)$ (*vote share*

[11] Keep in mind that policy-motivation does not exclude the possibility that a Partisan is also office-motivated, i.e., cares about the policy outcome as well as the rent he can extract when holding office.

[12] Cox (1990) follows a similar approach but extends the analysis to elections for multi-member districts, including the proportional representation system. For the purpose of our survey we focus only on single office elections. Hence, it is Cox (1987) that is immediately relevant for us.

[13] That the set of feasible actions for each candidate is $X$ and not $X \cup \{\emptyset\}$ means there is no option to not be part of the race. This is a common feature of the *exogenous* candidacy models. Alternatively, one could model the set of feasible actions as $X \cup \{\emptyset\}$ but then set $\delta(\emptyset) > 0$ and $\delta(x) = 0$ for $x \in X$. This ensures that no candidate chooses action $\emptyset$.

*maximizer*) or his plurality $v_i(e) - \max_{k \neq i}\{v_k(e)\}$ (*plurality maximizer*).[14] As for the rest, the model corresponds to the unifying model described in Sect. 2 above.

Cox first studies the properties of pure-strategy Nash equilibria under the plurality rule. He shows that, in multi-candidate races (i.e., when the number of candidates $M \geq 3$), equilibria, if they exist, must be non-centrist and the degree of non-centrism must be substantial in the sense that at least one candidate must locate outside the 25–75 % quantile range of voters' ideal policies. Furthermore, Cox shows that the existence of equilibria cannot be generically guaranteed. Cox (1987, 1990) concludes that, under the plurality rule "[m]ulticandidate equilibria either do not exist, or they are decidedly non-centrist."

Cox then extends his analysis to cover the class of rank scoring rules. Under a (normalized) rank scoring rule, each voter submits a strict rank ordering over the $M$ candidates and the candidate ranked in the $j$th position is given a score $s_j \in [0, 1]$ such that $s_j \geq s_{j+1}$ for all $j = 1, \cdots, M-1$, with the normalization restrictions that $s_1 = 1$ and $s_M = 0$.[15] The election winner is the candidate whose total score is the biggest. A wide variety of different voting procedures can be considered as special cases of a rank scoring rule. For example, $s_1 = 1, s_j = 0$ for all $j \geq 2$ is equivalent to the plurality rule; $s_j = \frac{M-j}{M-1}$ is equivalent to the Borda count; $s_1 = \ldots = s_{M-1} = 1$, $s_M = 0$ is equivalent to the negative vote rule.

To compare rank scoring rules, Cox focuses on symmetric pure-strategy Nash equilibria, which he calls convergent equilibria (since, in these equilibria, all candidates adopt the same position). The extent of polarization under convergent equilibria can be parametrized by $|\alpha - 1/2|$ where $\alpha$ denotes the quantile of the distribution of voters' ideal policies at which the candidates are located. To characterize the convergent equilibria, we define *Cox's threshold of diversity* (see Myerson 1999), which is defined by

$$\tau(s, M) = \frac{1}{M} \sum_{i=1}^{M} s_i$$

Thus, Cox's threshold of diversity, $\tau(s, M)$, corresponds to the average score under rank scoring rule $s$ in a $M$-candidate race. For example, $\tau(s, M) = \frac{1}{M}$ for the plurality rule; $\tau(s, M) = \frac{1}{2}$ for the Borda count; $\tau(s, M) = \frac{M-1}{M}$ for the negative vote rule. This threshold specifies the largest possible share of the electorate, the preferences of which can be disregarded by all candidates.

[14] Cox also considers the possibility that the candidates have a more complicated objective which he calls *complete plurality maximization*, meaning a candidate, for a given margin of victory, seeks to maximize the plurality relative to other candidate(s). Cox's results generally do not depend on which of the objectives described above are candidates' objectives, hence he collectively refers to them as "admissible candidate objectives".

[15] The normalization is without loss of generality.

Cox (1987) states the following:

**Proposition 1** *Let $Q_\alpha$ denote the position of the $\alpha-$quantile of the distribution of voters' ideal policies. Then, a convergent equilibrium with $e = (Q_\alpha, \cdots, Q_\alpha)$ exists if and only if $1 - \tau(s, M) \leq \alpha \leq \tau(s, M)$.*

Proposition 1 tells us both whether or not a convergent equilibrium exists, and the degree of candidate polarization under the equilibrium. The family of rank scoring rules can be divided into three classes depending on their threshold of diversity:

1. The rules with $\tau(s, M) < 1/2$, which are called best-rewarding since they correspond to rules where scores tend to decrease faster at the top than at the bottom. Proposition 1 implies there is no convergent equilibrium under these rules. The plurality rule is best-rewarding in multi-candidate races.
2. The rules with $\tau(s, M) = 1/2$, for which there is a unique convergent equilibrium, with all candidates located at the median voter's ideal policy. The Borda count belongs to this class of rules.
3. The rules with $\tau(s, M) > 1/2$, which are called worst-punishing since they correspond to rules where scores tend to decrease faster at the bottom than at the top. Proposition 1 implies there is a continuum of convergent equilibria under these rules, with all candidates located at the same position in the interval $[Q_{1-\tau(s,c)}, Q_{\tau(s,c)}]$, which includes the median voter's ideal policy. The negative vote rule is worst-punishing in multi-candidate races.

It is interesting to observe that in two-candidate races ($M = 2$), all rank scoring rules have a threshold of diversity equal to $1/2$. Proposition 1 thus implies that under any rank scoring rule, a two-candidate race yields a unique convergent equilibrium at the median voter's ideal policy, thereby deriving the Median Voter theorem as a special case.

Cox (1987) provides a similar result for the class of non-rank scoring rules, i.e., the scoring rules where a voter can cast more than one vote but does not provide a ranking of candidates. A prime example of such a procedure is approval voting where the voter can vote for as many candidates as she wishes, and the candidate who gets the biggest vote total is elected.

## 4 Rationale Behind Endogenizing Candidacy

In this section we discuss shortcomings of Cox's (1987) approach that might be addressed by endogenizing candidacy.

### *4.1 Lack of Satisfactory Micro-Foundations*

Cox (1987) considers a set of candidates that is both exogenous and kept fixed across voting procedures. These two assumptions are arguably questionable. Indeed,

it is realistic to think that the decision to contest an election, especially a political election, is highly strategic. It is reasonable as well to think that the incentives for contesting an election are specific to a voting procedure and, therefore, that the set of candidates is not fixed across voting procedures but varies from one voting procedure to another. Ignoring the differential incentives for candidates to contest an election might therefore lead to erroneous predictions. A more complete theory of comparative voting procedures would endogenize candidacy and take account of the differences in strategic candidacy behavior across voting procedures.

Dutta et al. (2001) investigates strategic candidacy behavior, underlining the importance of endogenizing candidacy when studying and comparing the theoretical properties of voting procedures. The following simple example illustrates the strategic candidacy behavior considered by Dutta, Jackson and Le Breton.

*Example 1* Consider an election with three candidates (labeled a, b and c) and three voters (labeled 1, 2 and 3). Voters' preference orderings over the set of candidates are given by

$$\begin{cases} aP_1bP_1c \\ bP_2cP_2a \\ cP_3aP_3b \end{cases}$$

where $P_n$ is voter $n$'s strict preference relation. Candidate a (resp. b and c) shares the same preferences as voter 1 (resp. 2 and 3). The election is held under the plurality rule with ties broken lexicographically (i.e., candidate a wins in a tie against any other candidate, and candidate b wins in a tie against candidate c). To further simplify the example, we assume that voters are sincere. In these circumstances each candidate receives exactly one vote (candidate a from voter 1, candidate b from voter 2, candidate c from voter 3) and candidate a is elected. However, the election of candidate a is the worst possible outcome for candidate b who prefers himself to candidate c, and candidate c to candidate a. Now, candidate b would be strictly better off withdrawing from the race since candidate c would then receive a majority of votes (the votes of voters 2 and 3) and be elected, an outcome that candidate b prefers to the election of candidate a. □

While this example is highly specific, Dutta, Jackson and Le Breton establish that the type of strategic candidacy behavior illustrated in this example applies to most standard voting procedures. Specifically, they consider voting procedures that are unanimous and candidate stable.[16] A voting procedure is unanimous if a candidate

[16]They define formally a voting procedure as a function $V : 2^{\mathcal{M}} \setminus \{\emptyset\} \times \mathcal{P} \to \mathcal{M}$, where $\mathcal{P}$ is the set of all profiles of strict preference relations in which a candidate finds himself most preferred, such that for all $\mathcal{C} \in 2^{\mathcal{M}} \setminus \{\emptyset\}$ and $P \in \mathcal{P}$:

1. $V(\mathcal{C}, P) \in \mathcal{C}$,
2. $V(\mathcal{C}, P) = V(\mathcal{C}, P')$ for all $P' \in \mathcal{P}$ such that $P_n = P'_n$ for every $n \in \mathcal{N}$, where $P_n$ is a strict preference relation for voter $n$, and

is necessarily elected when he is the most-preferred candidate of *every* voter.[17] A voting procedure is candidate stable if no candidate can affect the election in a way that is beneficial for him by withdrawing from the race.[18] Dutta et al. (2001) shows that:

**Proposition 2** *If $\mathcal{N} \cap \mathcal{M} = \emptyset$, and a voting procedure $V$ is candidate stable and unanimous, then $V$ is dictatorial.*[19]

Thus, when potential candidates cannot vote, *every* nondictatorial and unanimous voting procedure is subject to strategic candidacy behavior.[20] Standard voting procedures are nondictatorial and unanimous.

In light of this result, Dutta et al. (2001, 1014–1015) concludes that "[t]he results here show that the outcome of *all* nondictatorial and unanimous voting procedures will be influenced by the entry decisions of candidates. This implies that it is not valid to treat the set of candidates as fixed for any nondictatorial voting procedure. As most of what we know about voting procedures treats the set of candidates as fixed, our results suggest that these need to be revisited accounting for strategic candidacy. . . . [I]n order to make meaningful comparisons across voting procedures, strategic entry and exit effects must be understood."

It is important to observe though that while Dutta, Jackson and Le Breton show that strategic candidacy behavior *may* matter when comparing the theoretical properties of voting procedures, they do not establish that it *does* actually matter. In other words, they do *not* establish that the incentives for candidates to contest an election differ across voting procedures and/or that the comparative properties of voting procedures are different when candidacy is endogenous compared to when candidacy is exogenous. The literature discussed in Sect. 5 below answers this question, showing that strategic candidacy behavior varies across voting procedures and that it does matter for the comparative properties of voting procedures.

3. $V(\mathcal{C}, P) = V(\mathcal{C}, P')$ for all $P' \in \mathcal{P}$ such that $P\,|_{\mathcal{C}} = P'\,|_{\mathcal{C}}$, where $P\,|_{\mathcal{C}}$ is the profile of strict preference relations $P$ restricted to a non-empty set of candidates $\mathcal{C}$.

In words, a voting procedure chooses a candidate from the non-empty set of candidates $\mathcal{C} \subseteq \mathcal{M}$ (condition 1), is determined only by voters' preferences (condition 2), and depends only on the preferences over the set of candidates $\mathcal{C}$ (condition 3).

[17] Formally, a voting procedure $V$ is unanimous if $V(\mathcal{C}, P) = b$ for any $\mathcal{C} \subset \mathcal{M}$, $P \in \mathcal{P}$, and $b \in \mathcal{C}$ such that $bP_n c$ for all $c \in \mathcal{C} \setminus \{b\}$ and every voter $n \in \mathcal{N}$.

[18] Formally, a voting procedure $V$ is candidate stable if $V(\mathcal{M}, P) = V(\mathcal{M} \setminus \{a\}, P)$ or $V(\mathcal{M}, P)\, P_a\, V(\mathcal{M} \setminus \{a\}, P)$ for every $a \in \mathcal{M}$ and $P \in \mathcal{P}$.

[19] Formally, a voting procedure $V$ is dictatorial if there is a voter $n \in \mathcal{N}$ such that $V(\mathcal{M}, P) = top(\mathcal{M}, P_n)$ and $V(\mathcal{M} \setminus \{a\}, P) = top(\mathcal{M} \setminus \{a\}, P_n)$ for all $P \in \mathcal{P}$ and $a \in \mathcal{M}$, where $top(\mathcal{C}, P_n)$ is voter $n$'s most preferred candidate in candidate set $\mathcal{C}$.

[20] Dutta et al. (2001) and Eraslan and McLennan (2004) examine the robustness of the above result to relaxing each of the following three assumptions: (1) the set of candidates is disjoint from the set of voters; (2) preferences are strict; and (3) ties are broken deterministically.

## *4.2 Explaining Duverger's Propositions*

Duverger (1954) observes empirically that (1) the plurality rule favors a two-party system while (2) runoff and proportional representation favor a multi-party system. Riker (1982) dubs the first observation Duverger's law and the second observation Duverger's hypothesis. Maurice Duverger proposed two possible explanations for these empirical observations, a "mechanical effect" and a "psychological effect". The mechanical effect is related to the way a voting procedure mechanically converts votes into sets of election winners. The psychological effect is related to the strategic behavior of voters.

Unfortunately, Cox (1987) sidesteps explaining Duverger's propositions in two important ways. First, Cox takes the voting behavior to be sincere and the set of candidates to be exogenously given, thereby assuming away the psychological effect. In this way, he foregoes the chance of discriminating between the mechanical effect and the psychological effect as possible explanations for Duverger's propositions. Second, Cox restricts attention to symmetric pure-strategic equilibria, meaning that in equilibrium all candidates are located at the same position and tie for election. Thus, the equilibrium number of parties/candidates corresponds to the assumed number of candidates in the race. In other words, Cox assumes the party system; he does not derive it from the analysis.

Where can we go from here if we want to explain Duverger's propositions? One possible direction is to take account of strategic voting behavior. This approach is in line with explaining Duverger's propositions by the psychological effect mentioned above. This approach has been followed by, among others, Palfrey (1989), Myerson and Weber (1993), Myerson (2002) and Dellis (2013). Another possible direction, in line with the topic of this survey, is to take account of strategic candidacy behavior, by endogenizing candidacy. This approach is in the spirit of explaining Duverger's propositions by electoral barriers to exiting/entering the race. In our view this approach has the advantage over the previous one of requiring the coordination of a smaller number of players (candidates and party strategists instead of voters) who, furthermore, face higher stakes and are arguably more highly-strategic players. Finally, a third possible direction is to combine the two previous approaches, allowing both strategic voting behavior and strategic candidacy behavior. This approach has the advantage that it allows discriminating between these two types of strategic behaviors as possible explanations for Duverger's propositions.[21] We discuss in Sect. 5 below the latter two approaches.

[21] See Bol et al. (Forthcoming) for an attempt at using laboratory experiments to evaluate the contributions of strategic voting behavior and strategic candidacy behavior in explaining Duverger's law.

### 4.3 Allowing for Policy Motivation

Cox (1987) adopts the canonical Hotelling-Downs model in which candidates are purely office-motivated and can commit credibly to any policy position. However, it is difficult to imagine that in actual elections, candidates are not policy-motivated at all, i.e., do not care intrinsically about the policy outcome. A more satisfactory approach would take account of the policy-motivation of candidates.

However, accounting for the policy-motivation of candidates raises a dynamic consistency issue relative to candidates' commitment to policy positions. Alesina (1988) raises this issue, showing that a policy-motivated candidate has an incentive to announce any policy position that maximizes his electoral prospects and, once elected, to renege on his promise and adopt the policy he prefers rather than the policy he promised. When making their voting decision, rational voters will account for this incentive, thereby preventing a candidate from committing credibly to policy positions other than his most preferred policy. The inability of candidates to commit credibly to policy positions has received empirical support from Lee et al. (2004).

This discussion suggests that if we relax the assumption that candidates are purely office-motivated and introduce policy-motivation, then we must concomitantly relax the assumption of credible commitment. But if we consider candidates who both are policy-motivated and are unable to commit credibly to policy positions other than their most preferred policy, we must rationalize the policy preferences of the candidates; we cannot simply assume the policy preferences of candidates but must explain which types of policy preferences are represented in the set of candidates. A natural way to do so is by endogenizing candidacy. Specifically, one can consider a setting where, before the election, citizens decide strategically whether or not to contest the election. The citizen-candidate model discussed in Sect. 5 below adopts such an approach.

### 4.4 Equilibrium Characterization

As we have just discussed in the previous section, Cox (1987) adopts the Hotelling-Downs model of electoral competition. This model exhibits two important and well-known difficulties related to (pure-strategy Nash) equilibrium characterization.

1. **Equilibrium characterization in multi-candidate elections.** The simplest version of Hotelling-Downs model bypasses the difficulty of characterizing equilibria for multi-candidate elections by restricting the number of candidates to only two. In light of Duverger's law, this restriction is arguably justifiable when considering plurality rule elections. However, it is not justifiable when

comparing the theoretical properties of voting procedures. This is because all standard voting procedures are equivalent in the context of two-candidate races. In other words, assuming only two candidates would amount to eliminating all differences between voting procedures and would lead to the conclusion that all standard voting procedures exhibit the same theoretical properties, a conclusion that might not be robust to considering multi-candidate elections or to allowing them by endogenizing candidacy.

A meaningful comparison of the theoretical properties of voting procedures thus involves considering (in exogenous candidacy models) or allowing (in endogenous candidacy models) multi-candidate races. The problem though is that (pure-strategy) equilibrium characterization in the context of a Hotelling-Downs model with multiple candidates is notoriously difficult. To bypass this problem, Cox (1987) restricts attention to symmetric (pure-strategy) equilibria.[22] But this solution has several limitations on its own. One of these limitations is the incompleteness of equilibrium characterization, raising the question whether the properties identified in the context of symmetric equilibria generalize to the whole set of equilibria. Another of these limitations is that, by definition of a symmetric pure-strategy equilibrium, all candidates adopt the same position in equilibrium, which is not consistent with empirical evidence showing that candidates adopt polarized positions (e.g., Ansolabehere et al. 2001 for elections to the U.S. House of Representatives).

2. **Equilibrium existence.** A second difficulty related to equilibrium characterization in the Hotelling-Downs model concerns the issue of equilibrium existence. Even in the context of two-candidate races, pure-strategy equilibria of the Hotelling-Downs model almost never exist when the policy space is multi-dimensional (see, for instance, Duggan's 2006 survey). Moreover, even when an equilibrium exists, it is not robust to small perturbations in voters preferences. Comparisons of voting procedures using the Hotelling-Downs model have therefore been limited to settings with a unidimensional policy space. But even with a unidimensional policy space, equilibrium existence can still be an issue for some voting procedures in the context of multi-candidate races. This is the case for the plurality rule as shown, for example, in Cox (1987).

Endogenizing candidacy offers a possibility for overcoming these two difficulties. It is important to remember though that it is not the only possibility, that there are other ways to address these difficulties. Moreover, as we argue in Sect. 5 below, endogenizing candidacy is no panacea. Indeed, this approach has its own limitations, such as the possible existence of multiple equilibria.

[22] Cahan and Slinko (2012) provides an attempt at characterizing non-symmetric (pure-strategy) equilibria in scoring rule elections.

## 5 Models with Endogenous Candidacy

Models of electoral competition with endogenous candidacy differ along two main dimensions:

1. **Candidate motivation.** Models of electoral competition belong to two traditions, the Downsian tradition and the Partisan tradition. Models that belong to the *Downsian tradition* consider candidates who are purely office-motivated. Models that belong to the *Partisan tradition* consider candidates who are policy-motivated.
2. **Sequence of entry.** Some models treat all potential candidates as making their decisions simultaneously. Other models treat candidates in an asymmetric way, some candidates being already established in the political arena while others are new. Established candidates make their decisions first, while new entrants move second, after the established candidates.

Table 3 summarizes our classification of models of electoral competition with endogenous candidacy along these two dimensions.

The first family of models belongs to the Downsian tradition and treats candidates in an asymmetric way, with some established candidates facing a threat of entry by new candidates. These models look at the stability of the party structure (the number of candidates/parties contesting the election) and the effect that the threat of entry has on the positioning of established candidates/parties. Palfrey (1984) and Weber (1992, 1997) are the seminal contributions in this family. The second family of models belongs to the Downsian tradition as well but, in contrast to the first family, treats all potential candidates in the same way, with all potential candidates making their candidacy decisions simultaneously. These models are concerned with the issue of equilibrium existence. Osborne (1993) is the seminal contribution in this family. Finally, the third family of models considers simultaneous entry decisions, as the previous family of models does, but, in contrast to the other two families that belong to the Downsian tradition, this third family belongs to the Partisan tradition. These models look at the emergence of the party structure and the rationalization of candidates' policy preferences. Osborne and Slivinski (1996) and Besley and Coate (1997) are the seminal contributions in this family.

We now discuss each family of models in turn. For each family, we will outline the key features of the canonical model, discuss how it has been used for comparing the theoretical properties of voting procedures, and highlight the predictions they make on both the degree of polarization and the number of candidates contesting

**Table 3** Classification of models with endogenous candidacy

| | Downsian tradition | Partisan tradition |
|---|---|---|
| Sequential entry | Models of entry deterrence (family 1) | |
| Simultaneous entry | Hotelling-downs models with strategic candidacy (family 2) | Citizen-candidate models (family 3) |

the election. Bol et al. (2015) discusses in greater detail each family of models when applied to elections under the plurality rule.

## 5.1 Models of Entry Deterrence

The canonical model in this family is similar to the unifying model described in Sect. 2, except for the following two specificities:

- All (potential) candidates are Downsian, seeking to maximize their probability of winning the election and committing credibly to the policy they will implement if elected. A candidate enters the race if and only if he expects to be elected with a positive probability.[23]
- The defining feature of this family of models is its partitioning of the set of potential candidates $\mathcal{M}$ into two subsets. One subset contains two established candidates, which we call candidates L and R.[24] The other subset contains one potential entrant, which we call E. We write the set of potential candidates as $\mathcal{M} = \{L, R, E\}$. The two established candidates play a Cournot game vis-à-vis each other, making their candidacy decisions simultaneously and independently, while acting as Stackelberg leaders vis-à-vis the potential entrant, who makes his candidacy decision after the two established candidates have made theirs. Modelling electoral competition in this way, as a hierarchical game, captures the possibility for the two established candidates to deter successful entry by the potential entrant. Entry deterrence is the main focus of this family of models.

Thus, the policymaking process is modeled here as a four stage game. At the first stage, each established candidate $i\,(= L, R)$ makes independently his candidacy decision $x_i \in X \cup \{\emptyset\}$. At the second stage, the potential entrant E makes his candidacy decision $x_E \in X \cup \{\emptyset\}$ after having observed $x_L$ and $x_R$. At the third stage, voters observe the triplet $(x_L, x_R, x_E)$ and vote sincerely over the set of standing candidates. A voter's indifference between an established candidate and the entrant

[23]Contributions in this family differ in the assumptions they make on the entry cost and on candidates' objectives. Palfrey (1984) and Weber (1992) consider settings where there is no entry cost, with all potential candidates standing for election and seeking to maximize their vote share. Weber (1997) considers a setting in which candidates seek to maximize their vote share and a potential candidate contests the election if and only if he expects to receive at least a fraction $q \in [0, 1]$ of the votes. Finally, Callander (2005a,b) and Callander and Wilson (2007) consider settings in which candidates seek to maximize their winning probability and a potential candidate contests the election if and only if he anticipates to win with positive probability.

[24]To simplify exposition, in this survey we consider the case where there are only two established candidates. Weber (1997) and Callander (2005a,b) consider a version of the model with an arbitrary finite number $M \geq 2$ of established candidates.

is broken in favor of the established candidate. Finally, at the fourth stage, the elected candidate implements his position.[25]

In terms of comparison of voting procedures, this family of models has been used to compare the plurality rule with the runoff rules. The runoff rules involve a repeated process of candidate elimination. Some runoff rules, such as the plurality runoff rule (see footnote 4 for a description of this rule), involve repeated voting. Other runoff rules, such as the alternative vote rule, involve a single ballot-paper, but repeated counting.[26] The latter runoff rules are sometimes called instant runoff rules. Runoff rules are commonly used for political elections around the world (see Farrell 2001 for examples).

Recall from Sect. 4 that Duverger's hypothesis states that runoff rules favor a multi-party system. As illustrated in Table 1 above, France provides a good example of Duverger's hypothesis; France is holding its national elections under runoff rules and has a multi-party system. At the same time, Australia provides a counterexample to Duverger's hypothesis; Australia is holding elections to the House of Representatives under a runoff rule, but has a party system which is dominated by two main parties. In view of these observations, a theory that seeks to explain Duverger's hypothesis must be able to explain both the cases where it applies and the cases where it does not. Callander (2005b) proposes such a theory. In particular, Callander is able to explain why a two-party system (with polarized candidates) can be stable under a runoff rule. Formally,

**Proposition 3** *Consider an election under a runoff rule. For all* $x_L \in [\alpha, m]$*, where* $\alpha$ *solves* $F(\alpha) = 1 - 2F\left(\frac{\alpha+m}{2}\right)$*, a two-party equilibrium exists, in which:*

1. $x_L \leq m \leq x_R$ *and* $|x_L - m| = |x_R - m|$,
2. $x_E = \emptyset$*, and*
3. *candidates L and R are each elected with probability 1/2.*

Thus, Proposition 3 establishes the existence of equilibria in which 1) the two established candidates stand at two positions located symmetrically around the median voter's ideal policy $m$ (1) and tie for election (3), while the potential entrant chooses to not contest the race (2). Moreover, the threat of entry by the potential entrant puts an upper-bound on the degree of polarization between the two established candidates, so that the potential entrant, if he were to enter in-between

[25]To simplify exposition, we follow Callander (2005b) and assume that the density of voters' ideal policies $f$ is single-peaked and symmetric around the median voter's ideal policy $m$.

[26]For example, the alternative vote rule used for elections to the Australian House of Representatives requires every voter to cast a single ballot, in which she rank-orders all the candidates, from first to last. If one candidate is ranked first by a majority of voters, he is elected. Otherwise, the candidate who is ranked last on the greatest number of ballots is eliminated. Votes are then recounted using transformed ballots from which the eliminated candidate has been removed. A candidate is elected if he is ranked first on a majority of the transformed ballots. Otherwise, a second candidate is eliminated and votes are recounted using newly transformed ballots. The counting process is repeated until a candidate is elected.

the two established candidates, would finish third and be eliminated in the first round.

The key feature of runoff rules explaining the existence of such two-party polarized equilibria is that the candidate furthest from the median $m$ cannot win under a runoff rule since, even if he makes it to the last round, he will then be defeated by the other candidate still standing who, being closer to the median $m$, will be preferred by a majority of voters. This feature of runoff rules has three important implications:

1. The two established candidates must be located symmetrically around $m$; otherwise, the one established candidate who is furthest from $m$ would be elected with probability zero (being defeated in the last round, if he makes it), which would contradict that he maximizes his probability of winning.
2. Neither of the two established candidates has a *centrifugal incentive* to deviate to a position further from $m$ since, by doing so, he would be located further from $m$ than the other established candidate, implying that his probability of winning would then drop from one half to zero.
3. The threat of entry by the potential entrant eliminates the *centripetal incentives* for the established candidates to converge towards the median $m$. To see this, suppose candidate $L$ were to deviate and locate at $x_L + \epsilon \in (x_L, x_R)$ instead of $x_L$. This would induce the potential entrant to enter at a position $x \in (x_L, x_L + \epsilon)$ since he would then win outright. Indeed, as was mentioned above, $|x_R - x_L|$ is such that a candidate in-between, in this case candidate $L$, would be eliminated at the first round. Moreover, $|x - m| < |x_R - m|$ implies that the entrant would defeat candidate $R$ at the second round.

To sum up, the threat of entry can stabilize a two-party system under a runoff rule and can support polarization by eliminating the centripetal incentives for the two established candidates to converge towards the median.[27]

By contrast, under the plurality rule the threat of entry leaves the centripetal incentives unabated. In other words, the third implication above does not apply to the plurality rule. To see this, consider again the case where candidate $L$ deviates and locates at $x_L + \epsilon$ for $\epsilon > 0$ small. If the potential entrant were to enter and contest the election at $x \in (x_L, x_L + \epsilon)$, he might be able to defeat candidate $L$, but he cannot defeat candidate $R$ who would then win outright. This is because the entrant would capture votes from candidate $L$, but not from candidate $R$. Candidate $R$ would then receive (almost) half of the votes, while the other two candidates would split the other half. Candidate $L$ can thus deviate and locate closer to $m$, capture votes from candidate $R$ and still deter successful entry. Hence, under the plurality rule, the threat

[27]It is important to keep in mind that this analysis provides a *possible* explanation for the stability of a two-party system under a runoff rule. There are other possible explanations such as ethnic composition (e.g., Ordeshook and Shvetsova 1994) or the specificities of each runoff rule. Discriminating between these different possible explanations is an important question which is not addressed in this analysis.

of entry does not eliminate the centripetal incentives of the established candidates to converge towards the median.

To sum up, this family of models highlights the following key difference between the plurality rule and the runoff rules: the threat of entry eliminates the centripetal incentives for established candidates to converge towards the median and stabilize a two-party system when the election is held under a runoff rule, but not when the election is held under the plurality rule. This happens because if an established candidate deviates towards the median, the entrant can eliminate both established candidates under a runoff rule, thanks to the multiple rounds, eliminating one established candidate at a time (the deviator at the first round and the other established candidate at the second round). This is not possible under the plurality rule where there is a single round.

While Callander (2005b) highlights an interesting difference between the plurality rule and the runoff rules, the analysis exhibits several limitations relative to explaining polarization. First, there is a continuum of equilibria in runoff rule elections, including one in which the two established candidates locate at the median $m$. Hence, the model can explain indiscriminately both convergence and polarization. Second, if the model can predict polarization in runoff rule elections, it cannot predict the polarization observed under the plurality rule. This is because Callander (2005b) considers a relatively high entry cost, with a candidate entering the race only if he expects to be elected with positive probability. This high entry cost insulates the established candidates from the threat of entry in plurality rule elections as long as they have not fully converged towards the median. Weber (1997) shows that for intermediate values of the entry cost (with a candidate entering the race if and only if he expects to obtain a vote share of at least $q \in [Q, 1/2]$ for some $Q > 0$) entry deterrence generates a stable two-party system with polarized candidates under the plurality rule as well. In this case, the threat of entry on the flank is serious and generates centrifugal incentives that counterbalance the centripetal incentives, and prevent full convergence by the two established candidates even under the plurality rule. A similar prediction is obtained by Callander (2005a) in the context of multi-district elections under the plurality rule, and by Callander and Wilson (2007) in the context of single-seat elections but with voters' abstention due to alienation.

## 5.2 *Hotelling-Downs Models with Strategic Candidacy*

The canonical model in this family is similar to the model described in Sect. 2, with a finite number $M \geq 3$ of potential Downsian candidates seeking to maximize their probability of winning and committing credibly to the policy they will implement if elected. A potential candidate enters the race if and only if he expects to be elected with a positive probability. Thus, this model differs from Cox (1987) in that candidacy is here endogenous instead of exogenously given. Also, this model differs from the above model of entry deterrence in that all potential candidates are here ex

ante identical; the model here makes no distinction between established candidates and potential entrant.

Contributions belonging to this family of models have compared three types of voting procedures: the plurality rule, the plurality runoff rule and the multiple vote rules.

### 5.2.1 Plurality Rule

Osborne (1993) considers the case of the plurality rule and shows that:

**Proposition 4** *Consider an election under the plurality rule. We have that:*

- *If $M = 3$, then for any distribution of voters' ideal policies $F$, the game has no pure-strategy Nash equilibrium.*
- *If $M \geq 4$, then for almost any distribution of voters' ideal policies $F$, the game has no pure-strategy Nash equilibrium.*

Thus, Proposition 4 establishes that Cox's findings for multi-candidate plurality rule elections are robust to endogenizing candidacy. Specifically, it shows that when there are multiple potential candidates, pure-strategy Nash equilibria almost never exist under the plurality rule and, if an equilibrium exists, it is non-convergent. This follows because of the restrictions that centripetal and centrifugal incentives impose on the distribution of voters' ideal policies $F$. To see this, recall that a potential candidate contests the election if and only if he expects a positive probability of winning. This means that, in equilibrium, all standing candidates tie for first place. Let $\underline{x}$ (resp. $\overline{x}$) denote the leftmost (resp. rightmost) position at which a candidate is standing.

We start by observing that there must be at least two candidates standing for election. Indeed, if no candidate is standing for election, one potential candidate could then enter the race and win outright. If there is only one candidate standing for election, a second potential candidate could then enter the race and, at a minimum, tie for first place. In both cases, the assumption that a potential candidate contests the election if he expects a positive probability of winning would be violated.

We continue by observing that $\underline{x} < m < \overline{x}$, i.e., an equilibrium, if it exists, is non-convergent. Indeed, if all candidates contesting the election were adopting the same position (i.e., $\underline{x} = \overline{x}$), then either another potential candidate could enter on their flank and win the election outright, or one of the candidates could improve his probability of winning by deviating and adopting a position directly on the left or the right. Likewise, if all candidate positions were located on the same side of the median (i.e., $m \leq \underline{x} < \overline{x}$ or $\underline{x} < \overline{x} \leq m$), then one candidate could improve his probability of winning by deviating and locating at $m$.

We further note that the centripetal incentives to converge towards the median imply that there must be two candidates standing at $\underline{x}$ and two candidates standing at $\overline{x}$. Indeed, if there was only one candidate at $\underline{x}$, this candidate could increase his winning probability by deviating and adopting a position slightly closer to the

median $m$, capturing votes from his neighbor(s) while leaving unchanged the vote total of every other candidate; since our deviator was already tying for first place, he would now win outright. To eliminate these centripetal incentives, two candidates must be standing at $\underline{x}$.[28] By the same argument, two candidates must be standing at $\overline{x}$. Hence there must be at least four candidates standing, explaining why pure-strategy Nash equilibria do not exist when $M = 3$.

Finally, the left and right constituencies of $\underline{x}$ (resp. $\overline{x}$) must be of the same size, i.e., there must be as many voters on the left of $\underline{x}$ (resp. $\overline{x}$) as there are voters on the right of $\underline{x}$ (resp. $\overline{x}$) who are voting for a candidate at $\underline{x}$ (resp. $\overline{x}$); otherwise, one of the two candidates at $\underline{x}$ (resp. $\overline{x}$) would have a centripetal or centrifugal incentive to move slightly to the left or right of $\underline{x}$ (resp. $\overline{x}$), depending on whether the left constituency or the right constituency is the biggest, since he would then capture this whole constituency and win outright. As we shall see below, this restriction on left and right constituencies is key for the comparison of voting procedures. Osborne (1993) shows that this restriction imposes conditions on the distribution of voters' ideal policies $F$ that are satisfied for almost no distribution.[29]

### 5.2.2 Plurality Runoff Rule

Because they generate centripetal and centrifugal incentives that are different from those generated by the plurality rule, other voting procedures are not exposed to this equilibrium existence issue. Brusco et al. (2012) illustrates this point for the plurality runoff rule, establishing the existence of multiple pure strategy Nash equilibria under this rule. As in Callander (2005b), the distinctive feature of the plurality runoff rule compared to the plurality rule is that a candidate cannot win the election if he is located further from the median $m$ than all the other candidates. This feature eliminates the centripetal and centrifugal incentives causing the equilibrium existence problem under the plurality rule. We now explain why.

Consider the situation where all potential candidates enter the race at the median $m$. All candidates tie for first place and are equally likely to be elected.

1. This candidacy profile *does not* constitute an equilibrium under the plurality rule since one candidate would have a centrifugal incentive to move slightly to the

[28] At the same time, there cannot be more than two candidates standing at $\underline{x}$ since one of them could then win outright by deviating and adopting a position on the left flank or on the right flank of $\underline{x}$. Indeed, the deviator would then capture essentially half of the votes that were going to the candidates at $\underline{x}$, instead of receiving only one third or less of these votes. Since all candidates must be tying for first place, the deviator would then win outright, implying that the deviation is beneficial and contradicting that we are at an equilibrium.

[29] Bol et al. (2015) discusses alternative approaches that have been taken in order to ensure the existence of a pure-strategy Nash equilibrium under the plurality rule. To give an example, Feddersen et al. (1990) shows that when the voting behavior is assumed to be strategic, instead of sincere, an equilibrium exists under the plurality rule, in which all candidates standing for election adopt the position preferred by the median voter.

left or right of $m$. By doing so, the deviator would win outright since he would capture (almost) half of the votes, while the other candidates would split the other half.

2. By contrast, this candidacy profile *does* constitute an equilibrium under the plurality runoff rule since neither candidate has a centrifugal incentive to deviate and move to a position different from $m$. If a candidate were to do so, he would qualify for the second round but would then be defeated by the other standing candidate, who is located at $m$ and who is therefore preferred by a majority of voters.

Besides an equilibrium with all potential candidates standing at $m$, equilibria with candidates standing at two polarized positions do exist under the plurality runoff rule. To understand why, we now construct an equilibrium in which four candidates stand for election, two at a position $x_L$ and the other two at a position $x_R$, with $x_L$ and $x_R$ located symmetrically around $m$ and not too distant from each other. Each of the four candidates qualifies for the second round with an equal probability and is equally likely to be elected. Neither candidate has a centripetal incentive to move towards the median since he would be defeated at the first round, $x_L$ and $x_R$ being not too distant from each other. Moreover, neither candidate has a centrifugal incentive to move further away from the median since he would then be the furthest candidate from the median and would necessarily be defeated at the second round (if not already at the first round).[30] Key to observe is that in the case of the plurality runoff rule, the elimination of the centripetal and centrifugal incentives does not require that the left and right constituencies of a candidate position are of equal sizes. This contrasts with what Osborne finds for the plurality rule.

Thus, Brusco, Dziubinski and Roy essentially confirm Callander's (2005b) main conclusion, namely, that the plurality runoff rule can support a two-party system with polarized candidates. However, Brusco, Dziubinski and Roy treat all candidates identically, while Callander makes a distinction between established candidates and potential entrant. This means that Brusco, Dziubinski and Roy explain the *emergence* of a two-party system, not just its *stability* as Callander does. For the rest, Brusco et al. (2012) exhibits the same drawbacks as Callander (2005b), namely, equilibrium multiplicity and the indiscriminate explanation of both convergence and polarization, as illustrated by the two equilibria we have constructed here.

[30]Moreover, no other potential candidate wants to enter the race at another position, for the same reasons that neither of the four candidates wants to deviate. And no other potential candidate wants to enter at $x_L$ or $x_R$ since he would split the votes with the other two candidates at this position, implying these three candidates would be eliminated at the first round and the two candidates at the other position would be moving to the second round.

### 5.2.3 Multiple Vote Rules

Xefteris (2016) provides another example of voting procedures which, by generating centripetal and centrifugal incentives different from those generated by the plurality rule, are not exposed to the equilibrium existence issue identified in Osborne (1993). Specifically, Xefteris considers a whole family of voting procedures, called the multiple vote rules (also sometimes called non-rank scoring rules). Under a multiple vote rule each voter votes for (or is allowed to vote for up to) $k$ candidates and the candidate who receives the most votes is elected. Multiple vote rules differ in the (maximum) number of votes $k \in \mathbb{N}$ that a voter can cast. We call $k$-vote rule the multiple vote rule that allows voters to cast (up to) $k$ votes. Observe that this family of voting procedures includes, as a polar case, the plurality rule, which corresponds to the $k$-vote rule where $k = 1$.

Xefteris establishes that the plurality rule is the only multiple vote rule for which an equilibrium exists for almost no distribution of voters' ideal policies $F$. Formally,

**Proposition 5** *Consider the family of multiple vote rules. We have that:*

1. *If $k = 1$, then there is no equilibrium for almost every distribution of voters' ideal policies $F$.*
2. *$\Phi_k$ is non-degenerate if and only if $k \geq 2$, where $\Phi_k$ is the set of distributions of voters' ideal policies for which equilibria with two positions at which candidates are standing exist under the $k$-vote rule.*[31]

The first part of Proposition 5 follows straightforwardly from Osborne (1993). The second part is established by constructing equilibria with candidates standing at two positions located symmetrically around the median $m$.

The intuition underlying Proposition 5 is that the centripetal and centrifugal incentives put stronger restrictions on the left and right constituencies of a candidate position under the plurality rule than under the other multiple vote rules. This point is informally illustrated in the following example. Consider a candidacy profile with candidates standing at two positions, $x_L$ and $x_R$, that are located symmetrically around the median $m$. Suppose that at each of the two positions, there are $k + 1$ candidates standing under the $k$-vote rule. The centripetal incentives for a candidate at $x_L$ to move slightly closer to the median are eliminated if $\sigma_R \leq \sigma_L + \frac{k-1}{k}\sigma_R$, where $\sigma_L$ and $\sigma_R$ correspond to the left and right constituencies of $x_L$, respectively. This condition implies that the (approximate) vote total of the deviator would be smaller than the (approximate) vote total of each of the other candidates still standing at $x_L$. By the same argument, the centrifugal incentives for a candidate at $x_L$ to move slightly further away from the median are eliminated if $\sigma_L \leq \frac{k-1}{k}\sigma_L + \sigma_R$. Taken together, these two conditions impose the following restriction on the relationship between the size of the left constituency of $x_L$ and the size of its

[31] The equilibrium concept is here a refinement of pure-strategy Nash equilibrium. We refer the interested reader to Xefteris (2016) for more details on this and on the concept of genericity.

right constituency: $\frac{\sigma_R}{\sigma_L} \in \left[\frac{1}{k}, k\right]$. Key to observe is that this restriction is looser the bigger $k$ is. In particular, this restriction is satisfied for the plurality rule ($k = 1$) only if $\sigma_L = \sigma_R$, i.e., only if the left constituency and the right constituency are of equal sizes. As illustrated in this example, the specificity of the plurality rule is that a deviator is able to capture a whole constituency, left or right, something which is not possible under the other multiple vote rules where the other candidates still standing at the position the deviator has just deserted continue to receive votes both from the left constituency and from the right constituency.[32]

## 5.3 *Citizen-Candidate Models*

The third family of models, called the citizen-candidate models, allows for simultaneous entry by all potential candidates but assumes that the potential candidates are Partisans, i.e., that, just like the voters, potential candidates have policy preferences. Formally, potential candidates are here assumed to be voters (i.e., $\mathcal{M} \subset \mathcal{N}$), which contrasts with the previous families of models where potential candidates are Downsians, and where the set of potential candidates is disjoint from the set of voters (i.e., $\mathcal{M} \cap \mathcal{N} = \emptyset$). Moreover, given that potential candidates are here Partisans, and consistent with Alesina's (1988) argument discussed in Sect. 4.3, it is here assumed that a candidate cannot credibly commit to implementing upon being elected any policy except his ideal policy. Thus, the set of feasible actions for each potential candidate $i$ is $\mathcal{E}_i = \{x_i, \emptyset\}$, where $e_i = x_i$ if $i$ stands for election and $e_i = \emptyset$ if he does not contest the election. If potential candidate $i$ stands for election, he bears an entry cost $\delta > 0$. Voting behavior is either assumed to be sincere (as in Osborne and Slivinski 1996), or strategic (as in Besley and Coate 1997). The rest of the model is similar to the model described in Sect. 2.

Thus, this model differs from all previous models in that candidates are here Partisans instead of Downsians. Also, this model differs from Cox (1987) in that candidacy is here endogenous instead of exogenous, and it differs from the model of entry deterrence in that there is here no distinction between established candidates and potential entrant.

### 5.3.1 Citizen-Candidate Model with Sincere Voting

To clearly explain the mechanics underlying the citizen-candidate approach, we focus on a simplified variant of the citizen-candidate model *à la* Osborne and Slivinski. In particular, we assume that the rents from holding office are $\beta = 0$

[32]To be sure, this example is not rigorous; its only purpose is to illustrate the mechanism underlying the difference between the plurality rule and the other multiple vote rules. The interested reader can consult the formal proofs in Xefteris (2016).

(i.e., that candidates are purely policy-motivated) and that the density of voters' ideal policies $f$ is weakly single-peaked and symmetric around the median, $m$. We furthermore take a functional form for voters' utility function, assuming that $u^n(x) = -|x - x_n|$ (which does not affect the qualitative results).

We start by characterizing the set of equilibria under the plurality rule, partitioning the set into three subsets: (1) the subset of one-candidate equilibria, in which there is only one potential candidate standing for election; (2) the subset of two-candidate equilibria, in which two potential candidates contest the race; and (3) the subset of multi-candidate equilibria, in which more than two candidates contest the election.

**Proposition 6** *Consider an election under the plurality rule. We have that:*

1. ***One-candidate equilibria.*** *An equilibrium in which potential candidate $i \in \mathcal{M}$ runs unopposed exists if and only if each of the following two conditions holds:*

   *(a)* $\delta \geq |x_i - x_j|$ *for each* $j \in \mathcal{M}$ *with* $|x_j - m| < |x_i - m|$*; and*
   *(b)* $\delta \geq \frac{1}{2}|x_i - x_j|$ *for each* $j \in \mathcal{M}, j \neq i$*, with* $|x_j - m| = |x_i - m|$.

2. ***Two-candidate equilibria.*** *An equilibrium in which $i, j \in \mathcal{M}$, $i \neq j$, run against each other exists if and only if each of the following three conditions holds:*

   *(a)* $x_i < m < x_j$ *and* $|m - x_i| = |m - x_j|$*;*
   *(b)* $\frac{1}{2}|x_i - x_j| \geq \delta$*; and*
   *(c) one of the following three sets of conditions holds: (i)* $F\left(\frac{x_i+m}{2}\right) > 1/3$*; or (ii)* $F\left(\frac{x_i+m}{2}\right) = 1/3$ *and* $\delta \geq \frac{1}{3}|x_i - m|$*; or (iii)* $F\left(\frac{x_i+m}{2}\right) < 1/3$ *and* $\delta \geq |x_i - m|$.

3. ***Multi-candidate equilibria.*** *There is no equilibrium with three or more candidates running for election.* ‖

An appealing attribute of the citizen-candidate model is that it is not exposed to the equilibrium existence issue arising under the plurality rule in the Hotelling-Downs family of models. However, this attribute comes at a price, namely, equilibrium multiplicity as is clear from the above proposition and as was pointed out by Roemer (2003).

We now discuss the intuition underlying the above proposition.

In one-candidate equilibria, the candidate is located at a position sufficiently close to the median $m$ so that no other potential candidate wants to enter the race. This requirement is straightforwardly satisfied for any other potential candidate whose ideal policy lies further away from the median voter's ideal policy since he would be defeated by the candidate, if he chose to enter. A potential candidate whose ideal policy lies closer to the median would be preferred by the median voter, and thus a majority of voters, and would be elected outright. Similarly, a potential candidate whose ideal policy is as far away from the median would leave the median voter indifferent and would tie for first place. In these latter two cases, the potential candidate is deterred from entering the race if his expected utility gain

from implementing his ideal policy is smaller than the entry cost $\delta$ (conditions 1a and 1b).

In two-candidate equilibria, each candidate must tie for first place. Otherwise, the losing candidate would be better off not running since he would save the entry cost without changing the policy outcome. By the same logic, the two candidates must be standing at two different positions. Otherwise, one of them would be better off not running since he would save the entry cost without changing the policy outcome. Given that the two candidates must be tying for first place while standing at two different positions, their ideal policies must be located symmetrically around the median so that they split equally the votes and tie for first place (condition 2a). In a two-candidate equilibrium neither of the two candidates should be better off not running and no other potential candidate should want to enter the race. The former happens when the two candidates are sufficiently polarized, so that their expected utility gain from adopting their ideal policy exceeds the entry cost (condition 2b). The latter happens when the two candidates are close enough to each other so that a potential candidate entering in-between would be defeated (condition 2ci) or would face an entry cost that exceeds his expected utility gain from being elected and adopting his ideal policy (conditions 2cii and 2ciii). Observe that potential candidates with more extreme ideal policies are necessarily deterred from entering the race since they would split votes with the candidate on their side of the median, thereby triggering the outright election of the other, less-preferred candidate.

If a multi-candidate equilibrium were to exist, the leftmost candidate or the rightmost candidate (or both) would be better off not running since his votes would be transferred to his closest neighbor. This vote transfer would improve the electoral prospects of this neighboring candidate and worsen the electoral prospects of the other candidates. This rules out the existence of equilibria with three or more candidates running for election.

*Comparing the Plurality Rule with the Plurality Runoff Rule* An important contribution of Osborne and Slivinski (1996) is its ability to compare the equilibrium outcomes under the plurality rule with those under the plurality runoff rule (hereafter referred to as the runoff rule).

Osborne and Slivinski show that the set of one-candidate equilibria supported under the plurality and runoff rules is identical. This is so because the incentives of the running candidate and of any potential second candidate are identical under the two rules. Also, similar to the plurality rule, there are no multi-candidate equilibria under the runoff rule when the rents from holding office, $\beta$, are small.

Under the simplified variant of the model we consider in this survey, it can be shown that the set of candidate positions in any two-candidate equilibrium under the plurality rule is of the type $\{m-\varepsilon, m+\varepsilon\}$ where $\varepsilon \in [c, \overline{\varepsilon}_p]$ or $[c, \overline{\varepsilon}_p)$. The upper bound $\overline{\varepsilon}_p$ is derived by the condition that if the positions were polarized beyond this point, some potential candidate located at a position in between will want to enter and, upon entry, will win the election. Turning to the runoff rule, it is shown that the set of candidate positions in any two-candidate equilibrium is also of the type

$\{m - \varepsilon, m + \varepsilon\}$ where $\varepsilon \in [c, \overline{\varepsilon}_r]$ or $[c, \overline{\varepsilon}_r)$. The upper bound $\overline{\varepsilon}_r$ is derived by the condition that if the positions were polarized beyond this point, a candidate located at a position in between will want to enter and, upon entry, will come among the top two voter-getters and thereby advance to the second round and eventually win. It can be shown that $\overline{\varepsilon}_r \leq \overline{\varepsilon}_p$.[33] This establishes the following.

**Proposition 7** *Under the simplified variant of the citizen-candidate model à la Osborne and Slivinski (1996) with $\beta = 0$ and $f$ (weakly) single-peaked and symmetric around the median m, the set of equilibrium positions under the runoff rule supports weakly less polarization than under the plurality rule.*

While the above result provides a sharp comparison, the magnitude of the difference in the extent of polarization may not be significant; indeed, for the case of atom-less distribution it is of measure zero. Osborne and Slivinski also argue that their result is robust to small perturbations around (weakly) single-peaked and symmetric densities $f$. However, for a general shape of distributions there is no unambiguous comparison between the degrees of polarization under the two rules; the equilibrium positions under the runoff rule *may* be more polarized.

Turning to the issue of the number of candidates, we saw that under the simplified variant of the Osborne-Slivinski model developed above the number of candidates under either rule is either one or two. Hence, while the model supports Duverger's law, it fails to find support for Duverger's hypothesis. Turning to the more general case, Osborne and Slivinski show that a two candidate election is more likely under the plurality rule than under the runoff rule, and equilibria with three or four candidates are more likely to exist under the runoff rule relative to the plurality rule. However, the authors are not able to provide more general results, in particular for the case of equilibria with more than four candidates.

*Comparison with Other Voting Procedures* Dellis and Oak (2016) uses the citizen-candidate model with sincere voting to explore the comparative properties of a family of non-rank scoring rules, which they parametrize by the minimum number of votes a voter must cast $s$ ($\geq 1$) and the maximum number of votes a voter can cast $t$ ($\geq s$). They call this family of voting procedures the $(s, t)$ rules. The family of $(s, t)$ rules can distinguish between rules which allow partial abstention ($s < t$) and rules which do not ($s = t$); rules which allow partial abstention allow each voter not to cast all her votes, while rules which do not allow partial abstention force each voter to cast all her votes. Under each $(s, t)$ rule, the candidate receiving the biggest total score is elected. This broad family of voting procedures includes as special cases the plurality rule ($s = t = 1$), the negative vote rule ($s = t = c - 1$) and approval voting ($s = 1, t = c - 1$), where $c$ denotes the number of standing candidates.

Dellis and Oak first show that allowing citizens to cast multiple votes alters the electoral incentives of candidates. In particular, it creates an incentive for multiple

[33] Intuitively, the restriction that no potential entrant, upon entry, can be among the top two voter-getters is a stronger restriction than that such entrant be the top vote-getter.

candidates to run at the same or at similar positions; in the literature, such candidates are referred to as *clone candidates.* This possibility, of generating clone candidacies, does not occur under the plurality rule since under that rule the candidates running at the same or a similar position will split the votes and therefore reduce the chance of their ideal policy from being implemented. Secondly, the presence of clone candidates dampens the incentive for more centrist candidates to enter the race since such entrants are not assured of getting partisan votes as they are already "absorbed" by the partisan clones. Hence, the authors show that allowing multiple votes leads to multiple candidacies, thereby confirming Duverger's hypothesis and leading to greater polarization, which runs against the purported rationale provided by electoral reformists who favor allowing multiple votes. In a related paper, Dellis et al. (2016) shows that the alternative vote rule does not suffer from clone candidacy and therefore supports (weakly) less polarization relative to the plurality rule.

To sum up, the citizen-candidate model with sincere voting helps us focus on the differences in candidates' entry/exit incentives under different voting procedures. The literature finds weak support for Duverger's law as well as Duverger's hypothesis though the multiplicity of equilibria makes it difficult to make sharp predictions in this regard. In their original formulation, Duverger's propositions were explained by the strategic behavior of voters (the psychological effect discussed in Sect. 4.2). An interesting feature of the citizen-candidate model is that the mechanics of the Duvergerian outcomes is driven by strategic behavior of the *candidates rather than voters.* More interestingly, regarding polarization of candidate positions, the literature indicates that merely allowing voters to cast more votes does not lead to more centrist outcomes. Rather, whether, and to what extent, multiple votes generate clone candidacies has a bearing on the matter. Voting procedures such as the plurality runoff rule and the alternative vote rule, which do not encourage clone candidacy, support (weakly) less polarization relative to the plurality rule.

### 5.3.2 Citizen-Candidate Model with Strategic Voting

The other variant of the citizen-candidate model is due to Besley and Coate (1997). The mechanics of the model are similar to the one described above. The main difference is that the set of voters, $\mathcal{N}$, is finite and it is assumed that voting is strategic. Furthermore, it is assumed that voters use weakly undominated strategies. This refinement notwithstanding, strategic voting makes it possible to sustain equilibria driven by self-fulfilling voter coordination. To see this, consider a model similar to the one discussed in the section above but with a finite number of voters with their ideal points distributed uniformly over the interval $[0, 1]$.[34] Let's first look at the case of the two-candidate equilibria under the plurality rule. If voting were strategic, even the most extreme positions (here 0 and 1) are supported as

[34]I.e., suppose that there is one citizen at each of the locations $0, 1/N, 2/N, \cdots, 1$ for some large integer $N$.

equilibrium policy outcomes under the plurality rule. This outcome can be supported if, following entry by a centrist candidate, the voters in the set $[0, 1/2)$ would continue to vote for the candidate at 0, while the voters in the set $(1/2, 1]$ would continue to vote for 1. These voting strategies are admissible since they are weakly undominated.[35] In other words, voters are worried about *wasting their vote* on the centrist candidate whom they deem unviable, and this becomes a self-fulfilling prophecy. On the other hand, as we saw above, when voting is sincere there is an upper bound on the degree of polarization due to the constraint that if the positions of the incumbent candidates were farther apart, then a centrist candidate would enter and receive the centrist votes. In this example that we are considering, for a range of entry costs (specifically, $1/9 \leq \delta \leq 1/3$), the most extreme two-candidate equilibrium positions supported under sincere voting are $1/6$ and $5/6$.[36]

*Comparison with Other Voting Procedures* There is a set of papers that use the citizen-candidate model with strategic voting to compare different voting procedures. Dellis and Oak (2006) considers elections under approval voting. They show that approval voting generates more moderate outcomes relative to the plurality rule. The intuition is the following: under approval voting, the set of weakly undominated strategies requires that a voter must vote for his most preferred candidate and must not vote for his least preferred candidate. This in turn implies that candidate profile where two extremist candidates were running against each other may not be sustained as equilibrium due to a threat of credible entry by a centrist candidate. For instance, when voter ideal points are uniformly distributed as in the example above, if there were two candidates running at the 0 and 1 positions then a candidate at the median, situated at 1/2, upon entry will get at least 50 % of the votes (i.e., votes of the voters with ideal policy in the range $[1/4, 3/4]$) whereas the extremist candidates will get at most 50 % of the votes. Hence, the entrant will either be an outright winner, or will be in a three way tie. If the entry cost is sufficiently small (in particular, $\delta < 1/6$) the centrist candidate would enter the race.

Dellis (2009) considers the family of scoring rules, which includes the $(s, t)$−rules discussed above. Concerning the $(s, t)$-rules, it is shown that whether partial abstention is allowed (i.e., $1 \leq s < t$), or not (i.e., $s = t > 1$) matters to the outcomes. In particular, not allowing partial abstention allows supporting at least as much polarization as the plurality rule. As we saw above, due to the wasting-the-vote effect, the plurality rule supports a maximal degree of polarization when voting is strategic. However, Dellis shows that giving voters multiple votes need not reduce this effect, since more votes may be *soaked up* by multiple candidates entering at extreme positions. On the other hand, allowing partial abstention puts an upper bound on polarization for the same reason as we described above for the case of approval voting.

[35] Under the plurality rule, voting for any candidate *except the least preferred* candidate is a weakly undominated strategy.

[36] It can be verified that at $\delta \in [1/9, 1/3]$ no candidate located in between $1/6$ and $5/6$ would want to enter, nor either of the incumbent candidates would want to step down.

## 6 Discussion

We began our survey by citing two types of motivations, theoretical and empirical, for endogenizing candidacy in models of electoral competition. We now examine whether, and to what extent, the investment in endogenous candidacy has yielded returns on these fronts.

On the theoretical side, we know from the work of Dutta, Jackson and Le Breton that strategic candidacy behavior matters. But their work is silent on whether strategic candidacy behavior differs and/or has different implications across voting procedures. The models of electoral competition with endogenous candidacy show that strategic candidacy behavior does differ across voting procedures and that these differences are critical to generating different equilibrium outcomes. The differential incentives for clone candidacies, identified in citizen-candidate models, provide a clear illustration of the differences in strategic candidacy behavior and their implications. Similarly, Dellis and Oak (2006) illustrates, using a citizen-candidate model with strategic voting, differential *entry* incentives under approval voting and the plurality rule: it is shown that under the plurality rule, even if two candidates are located at extreme left and right positions, then a moderate candidate can be deterred from entering and contesting the race. This outcome is sustained by voters' self-fulfilling expectations about wasting-their-vote on such a moderate candidate. On the other hand, there is an incentive for such a candidate to enter under approval voting since such entrant would be guaranteed to gather sufficiently many centrist votes.

Continuing with the theoretical rationale for endogenous candidacy, we argued that *in principle* these models mark an improvement over exogenous candidacy since they provide a more satisfactory micro-foundation for the stability and/or emergence of different party/candidate configurations (in terms of their number and positions). However, some endogenous candidacy models, much like the exogenous candidacy models, fail to have equilibrium existence under some voting procedures. For instance, as discussed in the previous section, models in family 2 do not have equilibria under the plurality rule. While the citizen-candidate models (family 3) generically have equilibrium existence, they suffer from the problem of equilibrium multiplicity. In so far as the purpose of a model is to offer testable predictions regarding the comparison of voting procedures, neither equilibrium non-existence nor equilibrium multiplicity are desirable features.

On the empirical side, we saw that endogenous candidacy models are an improvement over the exogenous candidacy models in that they are able to provide microfoundations for Duverger's propositions and the cases where they do not apply. However, explaining some cases requires sometimes conditions that are quite restrictive.

Turning to the issue of policy polarization, one gets ambiguous results as well. Depending on the model used, one may conclude that multiple vote procedures support either more, less, or as much polarization. We saw from Cox's work that each voting procedure generates, to a varying degree, centripetal and centrifugal forces. The degree of polarization under a voting procedure depends on the relative strengths of the centripetal and centrifugal forces. While these forces work in Cox (1987) on the *intensive margin*, the models of endogenous candidacy add to the mix, or consider, another dimension, namely an *extensive margin*. Depending on the specific model, the possibility of candidate entry/exit works to accentuate or inhibit the centripetal and centrifugal forces. For instance, consider the model of entry deterrence in Callander (2005b). In this model, the standard Downsian incentives under a runoff rule exert a centripetal force, inducing each party to move closer to the median. However, the threat of entry on its flanks exerts a centrifugal force, preventing the parties from moving too close to the median. The result of these forces is the possibility of a polarized, two-party equilibrium under the runoff rule. On the other hand, using the citizen candidate model with sincere voting (e.g., Osborne and Slivinski 1996 and Dellis et al. 2016), we find that under a runoff rule it is the threat of entry by a centrist candidate that exerts a centripetal force, generating equilibria where candidates assume less polarized positions relative to the plurality rule.[37] On this count, Callander (2005b) performs better than the citizen-candidate models, making predictions that are consistent with the stylized facts presented in the introduction of this survey.

Of particular interest in this regard is Dellis and Oak (2007), which compares the plurality rule vis-à-vis approval voting using a model of endogenous candidacy which may be thought of as a hybrid of models of families 2 and 3. They assume that candidates are policy motivated (as in the citizen-candidate models of family 3), but are able to credibly commit to policy positions other than their ideal ones (as in models of family 2). They show that approval voting can lead to polarized equilibria. The intuition behind this result is that approval voting provides an incentive for clone candidacies. When two positions, distributed symmetrically around the median, are each populated by more than one candidate, a candidate is deterred from moving closer to the median out of the fear that he will lose the votes of more extreme voters. Hence, the centripetal incentives are diluted due to clone candidacy. The plurality rule, on the other hand, does not support clone candidacy and therefore generates a centripetal force leading to equilibrium outcomes at or close to the median position. Hence, this model predicts that approval voting supports a greater degree of polarization than the plurality rule does.

[37] In the citizen-candidate model, there exists only the *extensive margin* for candidate behavior, i.e., each candidate either chooses to run at his ideal policy or not be in the race. In this family of models the system *as a whole* exhibits centrifugal, or centripetal, tendencies in the sense that it induces candidates located at symmetric, and therefore mutually balancing, positions around the median to stand for election, where their positions are further apart from, or closer to, the median.

## 7 Conclusion

Models of endogenous candidacy offer a promising avenue for comparative analysis of voting procedures. In particular, they can provide a more satisfactory micro-foundation for emergence and stability of different configurations of political parties (in terms of their number and positions) observed across elections around the world. They also provide an avenue for experimental research by providing testable predictions regarding different voting procedures, including those which so far have never been used in real life elections. As yet, there is no one model of electoral competition with endogenous candidacy that can satisfactorily explain all the institutional features. As we discussed in the section above, different models provide different insights regarding candidates and voters behavior under alternative voting procedures, yet each has its own blind spots or weaknesses. As a whole, this class of models suffers from a multiplicity of equilibria. Another shortcoming of these models is their lack of a serious treatment of political parties.We briefly discuss some of the approaches to addressing the shortcoming of the existing models as well as works that lie outside the scope of the survey.

Different approaches have been taken to tackle the issue of equilibrium multiplicity (in citizen-candidate models). One approach that has yielded success, in terms of reducing equilibrium multiplicity is to introduce informational frictions (see for example, Grosser and Palfrey 2014; Messner and Polborn 2004; and Eguia 2007). Another approach consists in refining the solution concept for citizen-candidate models à la Besley and Coate (see for example, De Sinopoli and Turrini 2002; Dhillon and Lockwood 2002; and De Sinopoli 2004). However, these works typically focus on a particular voting procedure, usually the plurality rule; whether there exists a refinement that is capable of generating unique equilibrium predictions across a large class of voting procedures remains a topic for ongoing research.

The other criticism levelled at the models of electoral competition, including models with exogenous as well as endogenous candidacy, is the absence of political parties. In the literature reviewed in this paper, there was essentially no distinction between a party and a candidate. However, much of the political science literature, going back to the work of Maurice Duverger, takes the formation and evolution of political parties and issues surrounding them seriously. The need to articulate the relationship between candidates and political parties is particularly significant in the citizen-candidate models, where candidates are endowed with policy preferences. John Roemer points out forcefully the need to incorporate political parties into the analysis of electoral competition. Speaking about the citizen-candidate model, he writes: "...parties organize political competition, and discipline candidates to commit to policies which are, in general, not their ideal policies" (Roemer 2006, 423). Morelli (2004) and Levy (2004) are attempts at introducing parties in the citizen-candidate approach, where parties serve as a commitment device for candidates.

Models of endogenous candidacy, in particular the citizen-candidate models, have been used for examining issues other than the one we focused on in this survey. Closer to the topic of this survey, Morelli (2004), which we mentioned above, also compares voting procedures (using a citizen-candidate model), but with multi-member districts, comparing the plurality rule with proportional representation. Other examples are De Sinopoli and Iannantuoni (2007), which investigates the cases where Duverger's hypothesis does not apply under proportional representation, and Iaryczower and Mattozzi (2013), which compares numbers of candidates, campaign spending and polarization in proportional representation and in plurality rule elections.

Another interesting question that the endogenous candidacy approach is naturally amenable to is regarding the attributes of the candidates. For instance, what attracts people into politics—rents from office or policy considerations? With endogenous candidacy we can study endogenous emergence of Downsian or Partisan candidates rather than assuming them to be of one type or another (see for instance, Callander 2008 and Dziubinski and Roy 2013). One can also look at other attributes such as honesty or ability that get selected into political office (see for instance, Caselli and Morelli 2004; Messner and Polborn 2004; and Mattozzi and Merlo 2008). One promising area of research would be to examine how the selection of such attributes vary with the choice of the voting procedure governing elections.

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